THERMAL RESISTANCE OF CONTACT WITH OXIDIZED METAL SURFACES

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A model of an elementary heat channel is analyzed which simulates the zone of contact with an oxide film. An equation is derived which describes the increment of contact resistance due to the presence of an oxide film. The theoretical results agree closely with test data.

It has been noted in the course of several studies on contactive heat transfer [1-4] that the thermal resistance of metallic contacts increases during surface oxidation. Neither Soviet nor foreign sources, however, refer to any theoretical research concerning the effect of oxide films on the thermal resistance of contacts.

Most nearly complete information about the physical nature of the heat transfer through an interface with an oxide film can be obtained by analyzing an elementary channel with an interlayer which simulates an oxide film. The data can then be generalized and extended to the problem of real oxidized metal surfaces in contact.

We will consider one half of such an elementary heat channel with a metal substrate and an oxide film (Fig. 1a), equating the thermal fluxes and the temperatures at all points along the surface $0 < r < r_0$, $z = \delta$. The total thermal resistance, according to this model, is the sum of resistances in series which are due to constriction of thermal flux lines within the metal zone R_1 and within the oxide zone R_2, respectively:

$$R = R_1 + R_2. \tag{1}$$

Resistance R_1 will be expressed in terms of the resistance due to constriction of thermal flux lines from section area πr_0^2 at $z \gg \delta$ to section area $\pi r_{0_1}^2$ at $z = \delta$, the thermal flux assumed constant throughout, and resistance R_2 will be expressed in terms of the equivalent resistance due to constriction of thermal flux lines through a cylinder of material with a resistance identical to that of the metal substrate within the zone $0 < z < \delta$. Moreover, the constriction of thermal flux lines starts at section $z = \delta$ with an area $\pi r_{0_1}^2$ and ends at section $z = 0$ with an area πa^2.

The thermal resistance due to constriction of the thermal flux in the metal substrate is, according to [5],

$$R_1 = \frac{\Delta T_1}{Q} = \frac{1}{4\lambda_{M} r_{0_1}} \varphi_1. \tag{2}$$

The constriction factor, with the thermal flux assumed constant, can be approximated as

$$\varphi_1 = \frac{32}{3\pi^2}\left(1 - \frac{r_{0_1}}{r_0}\right)^{1.5}. \tag{3}$$

The zone which simulates the oxide film is in this model represented by a disk with a diameter $2r_{0_1}$ and a thickness δ.

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Heat enters uniformly through the disk face at $z = \delta$ across the area $\pi r_{0_1}^2$. Through the other disk face at $z = 0$ the thermal flux undergoes maximum constriction into a contact spot of area πa^2.

The solution of this problem reduces to integrating the Laplace equation in cylindrical coordinates

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{4}$$

with the following boundary conditions:

$$\left.\begin{aligned} &\text{for } z = 0 \text{ and } 0 < r < a \quad -\lambda_0 \frac{\partial T}{\partial z} = \frac{Q}{\pi a^2}, \\ &\text{for } z = 0 \text{ and } a < r < r_{0_1} \quad -\lambda_0 \frac{\partial T}{\partial z} = 0, \end{aligned}\right\} \tag{5}$$

$$\text{for } z = \delta \text{ and any } r \quad -\lambda_0 \frac{\partial T}{\partial z} = \frac{Q}{\pi r_{0_1}^2}, \tag{6}$$

$$\text{for } r = 0 \quad -\lambda_0 \frac{\partial T}{\partial r} = 0, \tag{7}$$

$$\text{for } r = r_{0_1} \text{ and any } z \quad -\lambda_0 \frac{\partial T}{\partial r} = 0. \tag{8}$$

We will represent function $T(r, z)$ in the form

$$T(r, z) = G(z)B(r). \tag{9}$$

Having found the partial derivatives, inserted them into (4), and then dividing (4) by GB, we obtain

$$\frac{d^2 B}{dr^2} + \frac{1}{r}\frac{dB}{dr} + \beta^2 B = 0. \tag{10}$$

$$\frac{d^2 G}{dz^2} - \beta^2 G = 0. \tag{11}$$

The method of power series, when applied to the solution of Eqs. (10) and (11), yields

$$\begin{aligned} B(r) &= A_1 J_0(\beta r), \\ G(z) &= D_1 \operatorname{Ch} \beta z - D_2 \operatorname{Sh} \beta z. \end{aligned} \tag{12}$$

We now insert solutions (12) into (9) and obtain

$$T = A_1 J_0(\beta r)[D_1 \operatorname{Ch}(\beta z) - D_2 \operatorname{Sh}(\beta z)]. \tag{13}$$

Using the trigonometric formula for the hyperbolic cosine of the difference between two angles, we can transform expression (13) for n roots as follows:

$$T = \sum_{n=1}^{\infty} A_n J_0(\beta_n r) \operatorname{Ch}[\beta_n(\delta - z)]. \tag{14}$$

Integrating Eq. (4) for $T = B(r)$ yields the expression $B = A_0 \ln r + A_0$, which under conditions (7) and (8) reduces to

$$B = A_0. \tag{15}$$

For $T = G(z)$, on the other hand, Eq. (4) under condition (6) becomes

$$G = \frac{Q}{\pi \lambda_0 r_{0_1}^2}(\delta - z). \tag{16}$$

Therefore, expression (14) may be rewritten as

$$T = A_0 + \frac{Q}{\pi \lambda_0 r_{0_1}^2}(\delta - z) + \sum_{n=1}^{\infty} A_n \operatorname{Ch}[\beta_n(\delta - z)] J_0(\beta_n r). \tag{17}$$

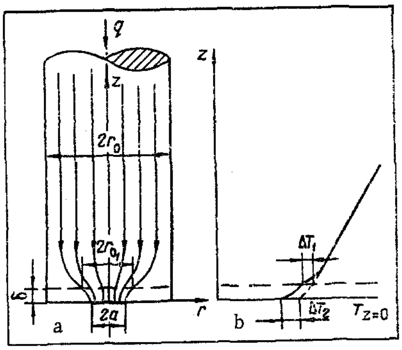

Fig. 1

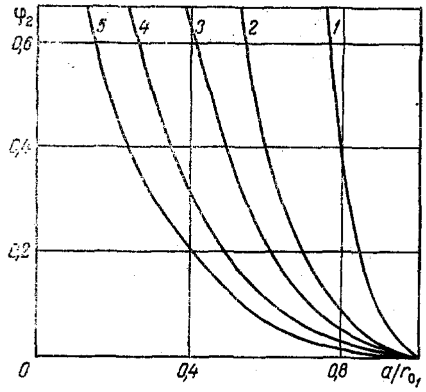

Fig. 2

Fig. 1. Model of an elementary heat channel (a) and temperature profile (b), for an oxidized contact.

Fig. 2. Constriction factor for the region with an oxide film, at various values of δ/a: 1) 0.01; 2) 0.05; 3) 0.10; 4) 0.20; 5) 0.30.

We will now expand condition (5) into a Fourier series of terms containing the Bessel function:

$$-\lambda_0\left(\frac{\partial T}{\partial z}\right)_{z=0} = \frac{Q}{\pi r_{0_1}^2} + \frac{2Q}{\pi a}\sum_{n=1}^{\infty}\frac{J_1(\beta_n a)\,J_0(\beta_n r)}{(\beta_n r_{0_1})\,r_{0_1} J_0^2(\beta_n r_{0_1})}. \tag{18}$$

Transforming expressions (14) and (18) reciprocally yields

$$A_n = \frac{2Q}{\pi\lambda_0 a}\cdot\frac{J_1(\beta_n a)}{(\beta_n r_{0_1})^2 J_0^2(\beta_n r_{0_1})\,\mathrm{Sh}(\beta_n\delta)}. \tag{19}$$

Inserting (19) into (17), we then transform the latter expression to

$$T = A_0 + \frac{Q}{\pi\lambda_0 r_{0_1}^2}(\delta - z) + \frac{2Q}{\pi\lambda_0 a}\sum_{n=1}^{\infty}\frac{\mathrm{Ch}\,\beta_n(\delta - z)\,J_1(\beta_n a)\,J_0(\beta_n r)}{\mathrm{Sh}(\beta_n\delta)\,(\beta_n r_{0_1})^2 J_0^2(\beta_n r_{0_1})}. \tag{20}$$

The contact resistance due to constriction of the thermal flux within the region of the oxide film is, on the basis of this model,

$$R_2' = \frac{\Delta T_2}{Q} - \frac{\delta}{\pi\lambda_0 r_{0_1}^2}. \tag{21}$$

The mean temperatures across areas πa^2 at $z = 0$ and $\pi r_{0_1}^2$ at $z = \delta$ are expressed as

$$T_1 = \frac{2\pi}{\pi a^2}\int_0^a T_{z=0}\,r\,dr,$$

$$T_2 = \frac{2\pi}{\pi r_{0_1}^2}\int_0^{r_{0_1}} T_{z=\delta}\,r\,dr.$$

Solving (21) with (20) and with the mean temperatures at those areas, we obtain the thermal resistance of a contact in the region of the oxide film

$$R_2 = \frac{4}{\pi\lambda_0 a}\varphi_2, \tag{22}$$

where

$$\varphi_2 = \frac{r_{0_1}}{a}\sum_{n=1}^{\infty}\frac{J_1^2(\beta_n a)}{\mathrm{th}(\beta_n\delta)(\beta_n r_{0_1})^3 J_0^2(\beta_n r_{0_1})}. \tag{23}$$

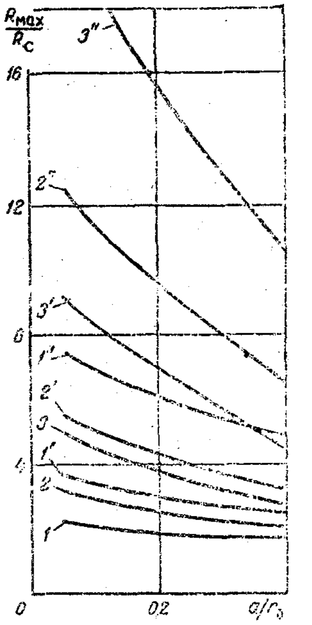

Fig. 3

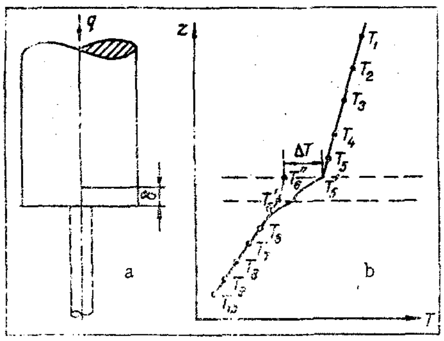

Fig. 4

Fig. 3. Ratio R_{max}/R_c as a function of the ratio a/r_0, for $\bar{\lambda}_M/\bar{\lambda}_0 = 10$ (curves 1-3), 20 (curves 1'-3'), 50 (curves 1"-3"), and for $\delta/a = 0.05$ (curves 1, 1', 1"), 0.1 (curves 2, 2', 2"), 0.2 (curves 3, 3', 3").

Fig. 4. Schematic diagram of a specimen simulating one half of an elementary channel with an oxide film in the contact zone (a) and the temperature profile along it (b).

The constriction factor φ_2 is shown graphically in Fig. 2, as a function of the ratio a/r_{0_1}, for various values of the parameter δ/a.

We may now rewrite Eq. (1) with (2) and (22), and thus obtain for the total resistance of a contact with oxide films

$$R = \frac{1}{2\bar{\lambda}_M a}\left(\frac{a}{r_{0_1}}\varphi_1 + \frac{16}{\pi}\cdot\frac{\bar{\lambda}_M}{\bar{\lambda}_0}\varphi_2\right). \tag{24}$$

Formula (24) is not suitable for calculating R, however, because r_{0_1} is a hypothetical quantity. Among all possible resistance values based on arbitrary choices of the thermal flux distribution, however, the maximum such value will obviously be closest to the actual one. Thus, for a given geometry of the channel components and for given values of $\bar{\lambda}_M$ and $\bar{\lambda}_0$, the choice of r_{0_1} within the range $a \le r_{0_1} \le r_0$ with which R becomes maximum will render the closest approximation to the true constriction resistance.

The relative increase in the thermal resistance of a contact due to the presence of an oxide film is approximated by the relation

$$\frac{R_{max}}{R_c} = \frac{1}{\psi}\left(\frac{a}{r_{0_1}}\varphi_1 + \frac{16}{\pi}\frac{\bar{\lambda}_M}{\bar{\lambda}_0}\varphi_2\right)_{max}. \tag{25}$$

The thermal resistance R_c of a single channel without an oxide film is expressed as

$$R_c = \frac{1}{2\bar{\lambda}_M a}\psi, \tag{26}$$

where the constriction factor ψ is a function of a/r_0 and can be calculated according to formula (3).

TABLE 1. Test Values and Calculated Values of R_{max}/R_c

a/r_0	Test data for δ/a				Theoretical data for δ/a			
	0,06	0,11	0,15	0,22	0,06	0,11	0,15	0,22
0,15	4,7	6,4	7,2	10,4	4,1	5,7	7	10,8
0,2	3,8	4,2	5,7	7,6	3,5	4,4	5,4	7,7
0,4	3	2,5	3,3	4,1	2,5	2,9	3,3	4,4

The maximum values of the term in brackets in (25) were calculated on a digital computer for each specific combination of ratios a/r_0, δ/a, $\overline{\lambda}_M/\overline{\lambda}_0$, and the selected value of r_{0_1}. The results of this computation, namely R_{max}/R_c as a function of a/r_0, is shown in Fig. 3 for various values of the parameter δ/a and three values of ratio $\overline{\lambda}_M/\overline{\lambda}_0$.

An analysis of the graphs in Fig. 3 indicates that the effect of an oxide film becomes stronger at higher ratios δ/a and $\overline{\lambda}_M/\overline{\lambda}_0$. Although these data do not represent exact solutions, they are useful for predicting and appropriately regulating the buildup of thermal resistance at a contact with oxidized surfaces.

In order to verify the validity of this proposed model, a prototype study was made on specimens simulating a heat channel with an oxide film in the contact zone (Fig. 4a). A cylindrical blank of grade M-2 copper was welded end-on-end to a cylinder of grade 1Kh18N9T stainless steel. Both cylinders were 80 mm in diameter and 70 mm long.

The steel portion of these composite cylinders was mechanically shaped into a form shown in Fig. 4a. Such test specimens served simultaneously as thermometers, for the purpose of which five Chromel–Alumel thermocouples had been mounted in each at various heights.

The temperature profile along a specimen surface is shown in Fig. 4b, from which the temperature drop ΔT due to constriction of thermal flux lines can be calculated.

Temperatures T_5' and T_6' are found by extrapolation of the curves, which have been plotted from the readings of thermocouples 1-5 and 6-10. Temperature T_6'' is determined from the thermal flux density, the temperature T_6, and the known thermal conductivity of grade 1Kh18N9T steel.

Specimens were tested with three different ratios a/r_0 (0.15, 0.20, 0.40) and four different ratios δ/a (0.06, 0.11, 0.15, 0.22) for each value of a/r_0. The ratio of thermal conductivities $\overline{\lambda}_M/\overline{\lambda}_0$ was maintained at the 23.8 level.

The tests were performed with an updated version of the apparatus in [4]. The thermal resistance of specimens simulating a metal–oxide junction was measured, as was that of specimen made of a single metal (copper) with analogous geometrical dimensions.

A comparison between test data and calculations according to formula (24) is shown in Table 1. The results indicate a satisfactory agreement between them. The widest discrepancy, up to 20-25%, occurs in the range of small ratios δ/a.

The correctness of the proposed model contributes to the feasibility of extending these results to multicontact junctions in joints between oxidized metal surfaces.

NOTATION

R is the thermal resistance of contact represented by a single channel, deg/W;

a is the radius of the contact area;

r_0 is the radius of the heat channel;

r_{0_1} is the hypothetical intermediate radius;

$\overline{\lambda}_M = 2\lambda_{M_1}\lambda_{M_2}/(\lambda_{M_1} + \lambda_{M_2})$,

$\overline{\lambda}_0 = 2\lambda_{0_1}\lambda_{0_2}/(\lambda_{0_1} + \lambda_{0_2})$ are the referred thermal conductivities, respectively, of the metal substrate and the oxide film in contact, W/m·deg;

Q is the thermal flux through a single channel, W;

δ is the thickness of an oxide film;

r, z	are space coordinates;
φ_1, φ_2, ψ	are the constriction factors referred to thermal flux lines;
β	are eigenvalues;
J_n	is the n-th order Bessel function.

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